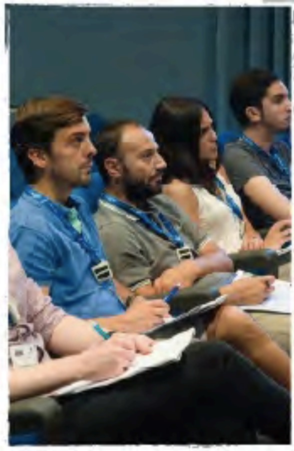
SUMMER SCHOOL

JULY
17-20
2018

POWERING THE INTERNET OF THINGS

PERUGIA, ITALY



School Topics

Fundamentals on energy, Linear and non-linear energy harvesting, Vibration harvesting, Thermoelectric energy harvesting, Energy storage, Micro power management, autonomous devices, Internet of Things, System integration, Applications to mobile devices.

Hands on session:
- Kinetic harvesting from real-world vibrations
- Wireless power transfer

To know more: www.nipslab.org/summerschool

NiPS Laboratory
Noise in Physical Systems

EnABLES

SUMMER SCHOOL

JULY
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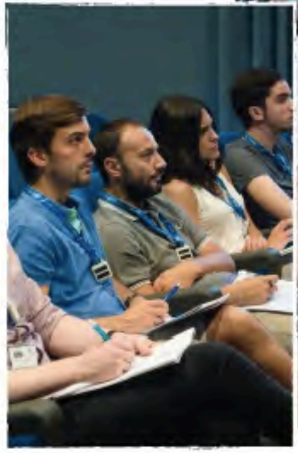
2018

ENERGY AWARE TRANS PRECISION COMPUTING

PERUGIA, ITALY



School Topics

Fundamentals on energy, Physics of Information, Transprecision algorithms, Adaptive-precision algorithms, Advanced compilation techniques, Integrating reduced precision concepts into applications and real code, Architecture and Circuits: RISC-V ISA/ Microarchitecture, Parallel Ultra Low-Power Processing ,PULP, FPGA, Approximate DRAM and memory controllers.

Hands on session:
- GAPUINO board/Mr. Wolf based board
- PULP

To know more: www.nipslab.org/summerschool

NiPS Laboratory
Noise in Physical Systems

OPRECOMP
Open Transprecision Computing

	July 16, Monday	July 17, Tuesday	July 18, Wednesday	July 19, Thursday	July 20, Friday
9:00 - 10:30		Fundamentals on energy	SW and TOOLS	Applications	Hands on session
		coffee break			
11:00 - 12:30		Physics of Information	SW and TOOLS	Applications	Hands on session
13:00 - 14:00		lunch	lunch	lunch	
14:00 - 14:30			Poster session		
14:30 - 15:00		Algorithms		Architecture and Circuits	Architecture and Circuits
14:30 - 16:00			Excursion + Social Dinner		
		coffee break		coffee break	
16:30-18:00		Algorithms		Architecture and Circuits	Architecture and Circuits
18:00-20:00	Registration and				

	July 16, Monday	July 17, Tuesday	July 18, Wednesday	July 19, Thursday	July 20, Friday
9:00 - 10:30		Fundamentals on energy	RF Energy Harvesting	Energy Storage 2	System Integration 1
		Coffee Break			
11:00 - 12:30		Linear Vibration Energy Harvesting	Energy Storage 1	Energy Storage 3	System Integration 2
13:00		Lunch	Lunch	Lunch	Lunch
14:00 - 14:30			Poster Session		
14:30 - 15:00		Non-linear Vibration Energy Harvesting		Micro Power Management 1	Hands On session
15:00 - 16:00			Excursion + Social Dinner		
		Coffee Break		Coffee Break	Coffee Break
16:30-18:00		Thermoelectric Energy Harvesting		Micro Power Management 2	Hands On session
18:00-20:00	Registration and welcome cocktail				
20:30					

EnABLES lectures program

Fundamentals on energy

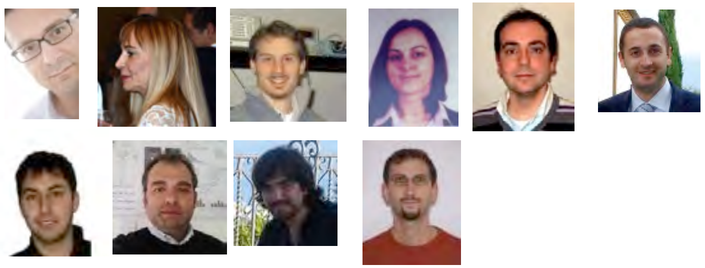
Luca Gammaitoni
NiPS Laboratory, University of Perugia

NiPS Summer school 2018, Perugia

NiPS Laboratory
Noise in Physical Systems

We are interested in energy transformation processes at micro and nano scales. Noise, fluctuations and zero-power computing.

2018 (9 FT, 1 PT)



FET Proactive projects

2006-2009 EC (SUBTLE VIFP)
2010-2013 EC (NANOPOWER VIIFP)
2010-2013 EC (ZEROPOWER VIIFP)
2012-2015 EC (LANDAUER VIIFP)
2013-2016 EC (ICT-Energy VIIFP)
2015-2018 EC (Proteus H2020)
2017-2020 EC (OPRECOMP H2020)
2018-2021 EC (ENABLES H2020)

University of Perugia (IT)

AD 1308

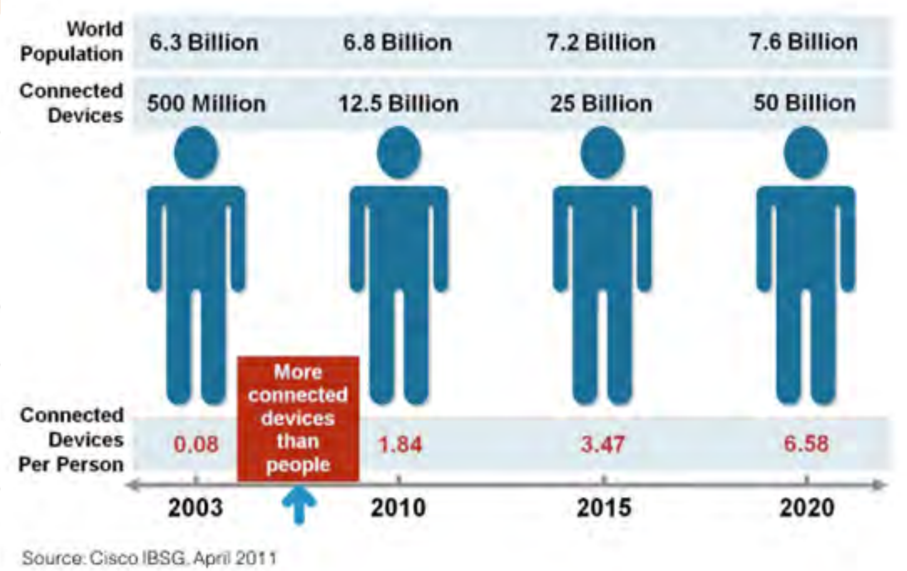
OPRECOMP
Open Transprecision Computing

EnABLES

April 4,
2005

March 13,
2013

A growing number of people accepted to join the so-called internet-of-things scenario



Before this scenario becomes a reality the device powering issue needs to be addressed and solved.

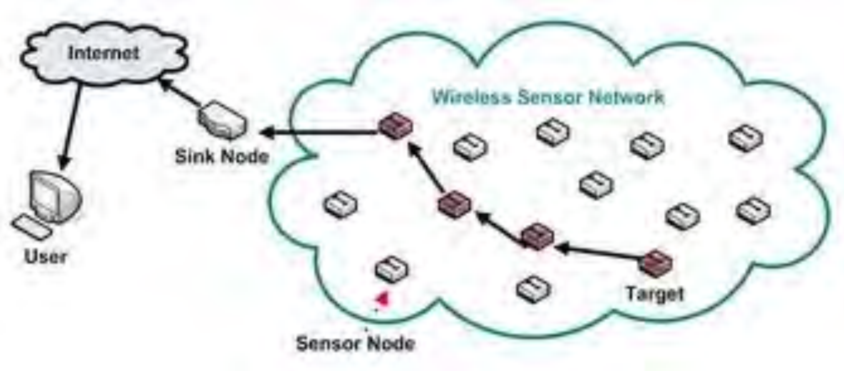
The challenge of efficient management of energy is a key aspect to consider in computing systems, especially for applications in smart sensors and Internet of Things devices.

European Commission **Workshop** on “Energy-Efficient Computing Systems, dynamic adaptation of Quality of Service and approximate computing”. Nov. 27 2014 - Brussels

Energy consumption has become a major issue for the future of ICT and robotics as well

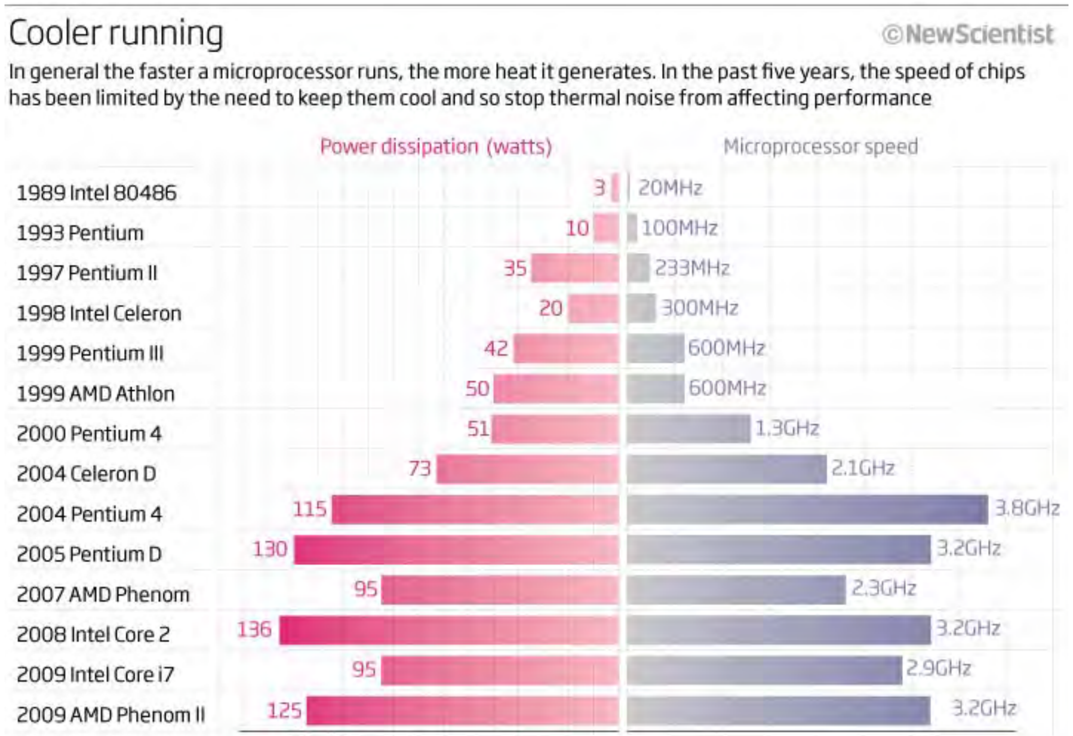
1) High performance computing systems

2) Autonomous microdevices, micro robots, wireless sensor networks



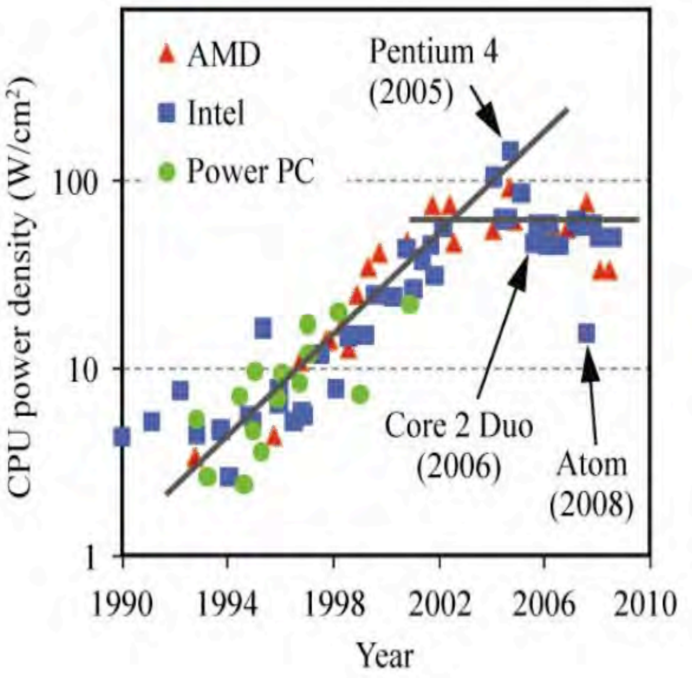
1) High performance computing systems

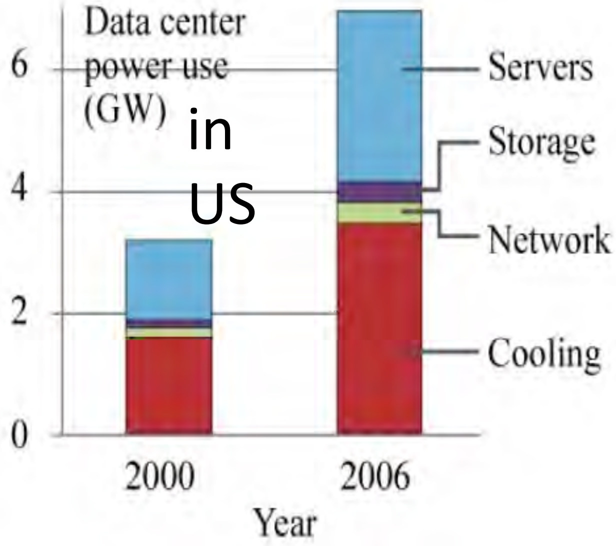
Energy consumption in computing systems has become a major issue for the future of ICT



ICT - Energy

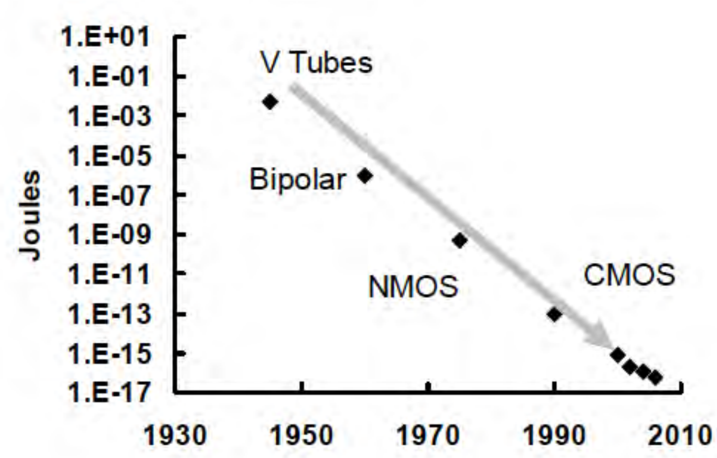
The binomial **ICT-Energy** has become the focus of future ICT research world wide





E. Pop, *Energy Dissipation and Transport in Nanoscale Devices*, Nano Res (2010) 3: 147–169

Energy required to operate microprocessors

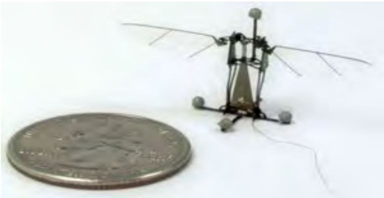


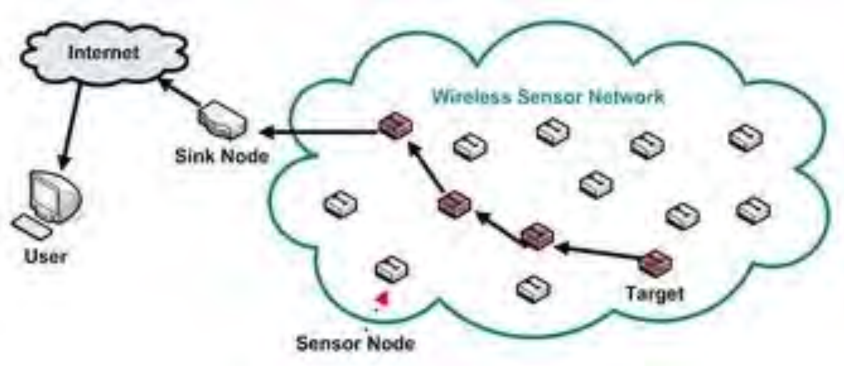
Shekhar Borkar, **Electronics Beyond Nano-scale CMOS,** Design Automation Conference, 2006 43rd ACM/IEEE

"...the resulting power density for these switches at maximum packing density would be on the order of 1MW/cm^2 – orders of magnitude higher than the practical air-cooling limit.."

Jeffrey J. Welser
The Quest for the Next Information Processing Technology , **2008**

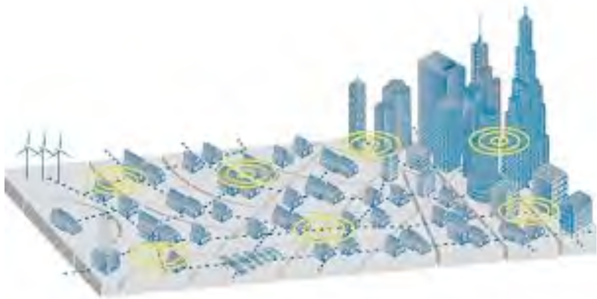
2) Autonomous microdevices, micro robots, wireless sensor networks

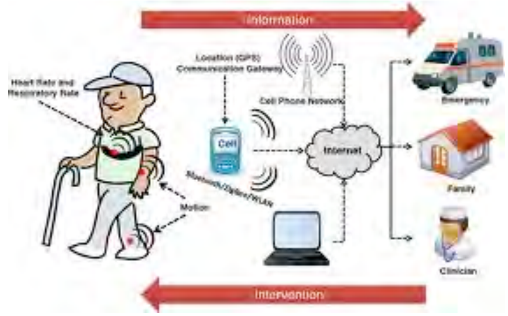




The promised land of ubiquitous computing

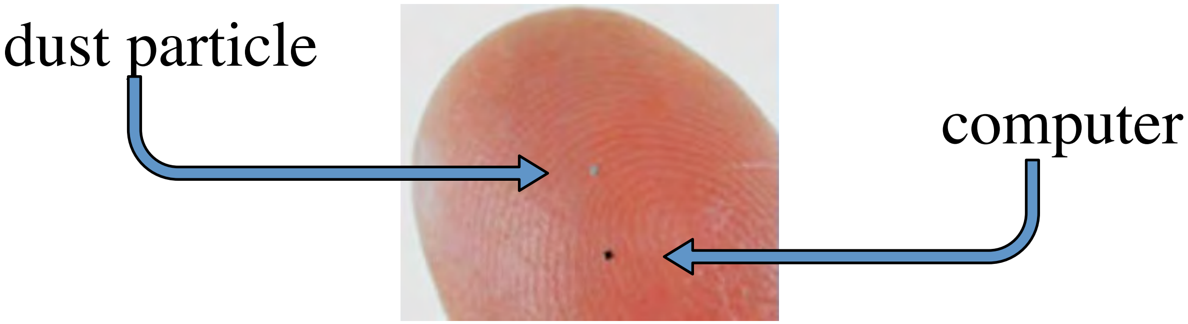
This is the land of wireless micro-sensors that continuously and ubiquitously measure, process and transmit data to improve our living.





This is the long-time announced revolution where the cities become smart and the human and animal health is monitored and controlled.

The promised land of ubiquitous computing

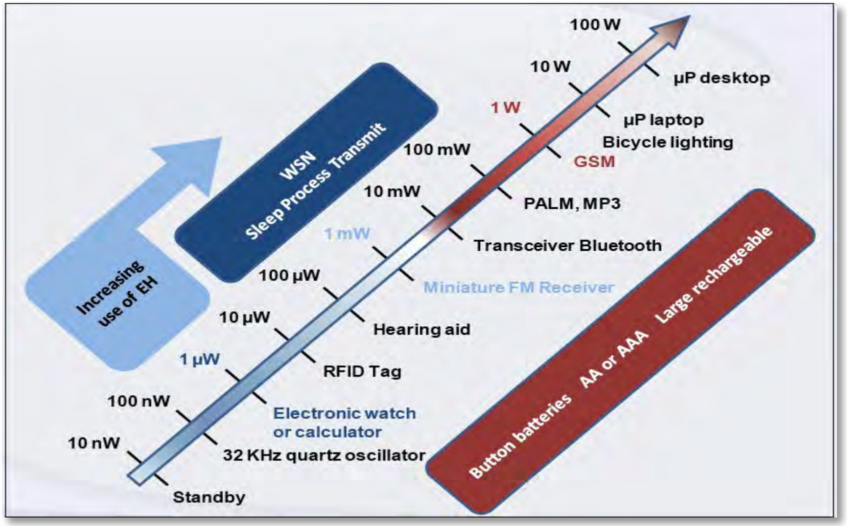


The land where computers are as small as dust particles

and bio-inspired robots appear

Why are we not there yet?

Energy required to operate the portable devices



Energy available from portable sources (energy harvesting)

Source: IDTechEx, "Energy Harvesting and Storage 2009-2019", Cambridge 2009. EH: Energy Harvesting; WSN: Wireless Sensors Network

Energy required to operate the portable devices

We need to bridge the gap by acting on both arrows

Energy available from portable sources

Energy required to operate computing devices

Why does it make sense to talk of a new field ICT-Energy ?

Energy available from the environment

"battery-free" devices

1) Energy dissipation in high performance computing systems
2) Powering autonomous microdevices, micro robots, wireless sensor networks

They both sits on a common scientific ground:
Micro and nano scale energy management

Questions like:

-How does electric energy get converted into heat at nanoscale
-How can we find an information transport solution that does not add to dissipation
-How can we harvest vibrations to power micro/nanoscale devices
-…

Could be asked and answered within this framwork.

ON A BROADER PERSPECTIVE

The well-known laws of heat and work transformation that lie at the base of the classical thermodynamics are going to **need a rethinking**. The very basic mechanism behind energy dissipation requires a new definition when non-equilibrium processes involving only few degrees of freedom are considered.

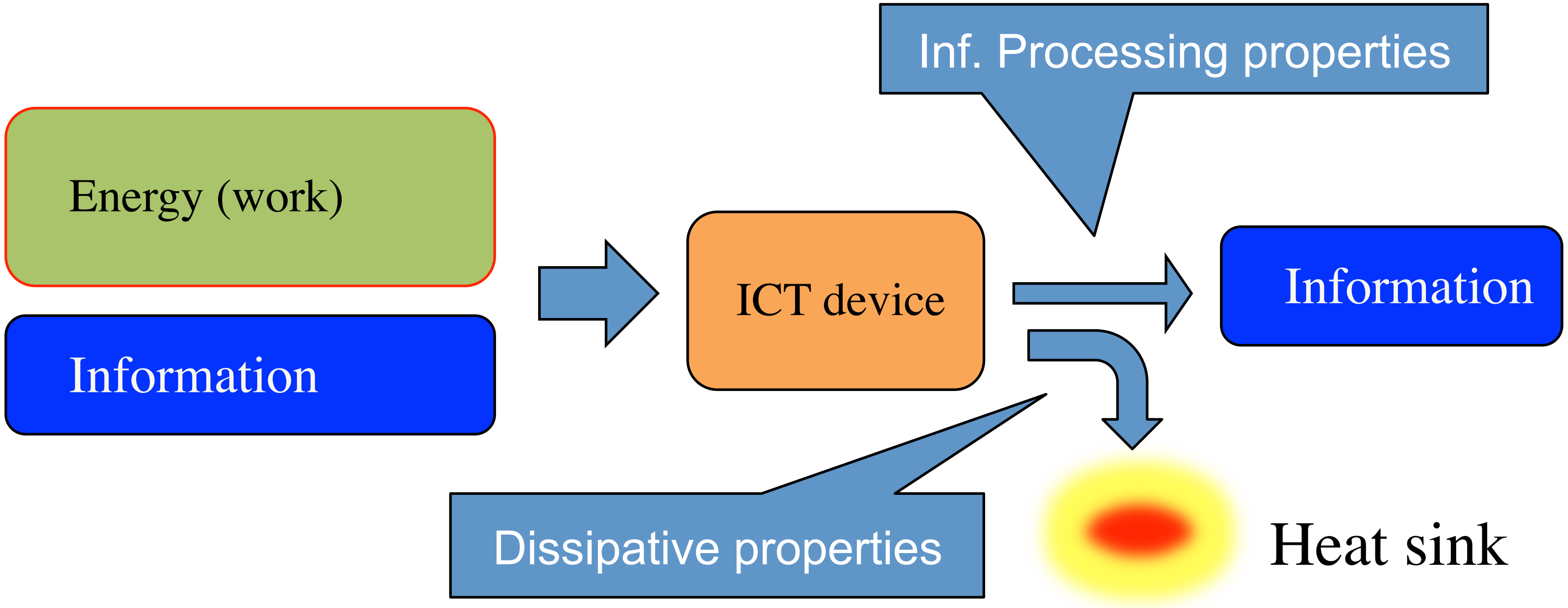
Industrial Revolution XVIII-XIX	ICT Revolution XX-XXI
Heat-Work relations	Fluctuation-Dissipation relations

CHALLENGE:

the description of **energy transformation processes at the nanoscale** aimed at unveiling new mechanisms for powering next generations of ICT devices.

A different approach to heat production: an ICT device is a special thermal machine

An **ICT device** is a machine that inputs information and energy (under the form of work), processes both and outputs information and energy (mostly under the form of heat).



L. Gammaitoni, There's plenty of energy at the bottom, Contemporary Physics, vol. 53, issue 2, pp. 119-135

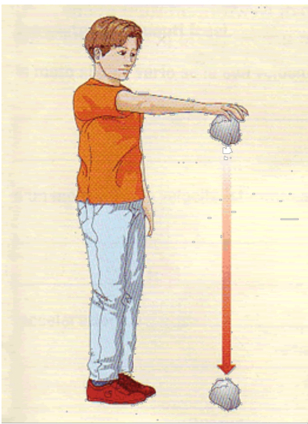
Energy

What is it?

Energy

In physics, energy (Ancient Greek: ἐνέργεια energeia "activity, operation”) is an indirectly observed quantity that is often understood as the **ability of a physical system to do work on other physical systems**

Capability of doing WORK… WORK = FORCE x Displacement



Energy

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is an indirectly observed quantity that is often understood as the
ability of a physical system to do work on other physical systems

Capability of doing WORK… WORK = FORCE x Displacement

Energy ?

Energy ?

Energy ?

Energy

"*It is important to realize that in physics today, we have no knowledge what energy is*."

Richard Feynman, in The Feynman Lectures on Physics (1964) Volume I, 4-1

Energy: a hint of history

The *vis viva* (living force), which **Gottfried Leibniz** defined as the product of the mass of an object and its velocity squared; he believed that total vis viva was conserved.

To account for slowing due to friction, Leibniz theorized that thermal energy consisted of **the random motion of the constituent parts of matter**, a view shared by **Isaac Newton**, although it would be more than a century until this was generally accepted.

Energy: a hint of history

The **conservation of energy** was proposed by Gottfried Leibniz over the period 1676–1689, the theory was controversial as it seemed to oppose the theory of conservation of momentum advocated by Sir Isaac Newton and René Descartes. The two theories are now understood to be complementary.

In 1807, **Thomas Young** was possibly the first to use the term "energy" instead of vis viva, in its modern sense.

Energy: a hint of history

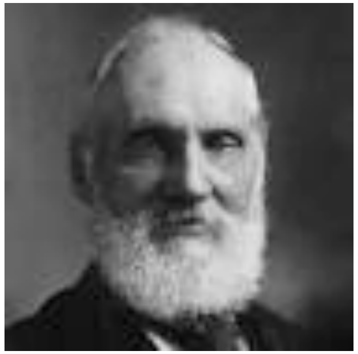
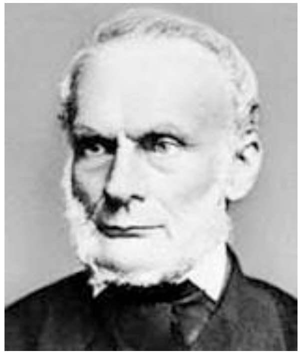
Gustave-Gaspard Coriolis described "kinetic energy" in 1829 in its modern sense.

In 1853, **William Rankine** coined the term "potential energy".

William Thomson (Lord **Kelvin**) amalgamated all of these laws into the laws of thermodynamics, which aided in the rapid development of explanations of chemical processes by

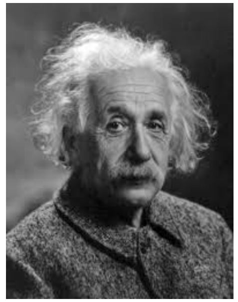
Rudolf Clausius, **Josiah Willard Gibbs**, and **Walther Nernst**.



Energy: a hint of history

It also led to a mathematical formulation of the concept of entropy by Clausius and to the introduction of laws of radiant energy by **Jožef Stefan**.



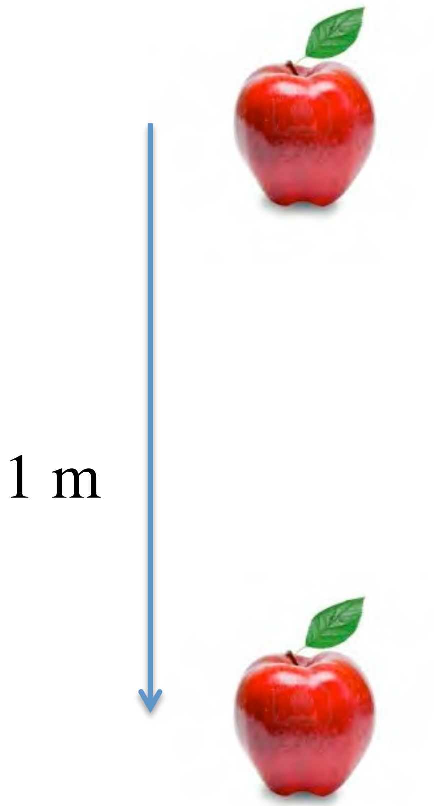
Albert Einstein proposed mass–energy equivalence in 1905 in a paper entitled "Does the inertia of a body depend upon its energy-content?”.

Since 1918 it has been known that the law of conservation of energy is the direct mathematical consequence of the translational symmetry of the quantity conjugate to energy, namely time (**Emmy Noether**).

Use of energy

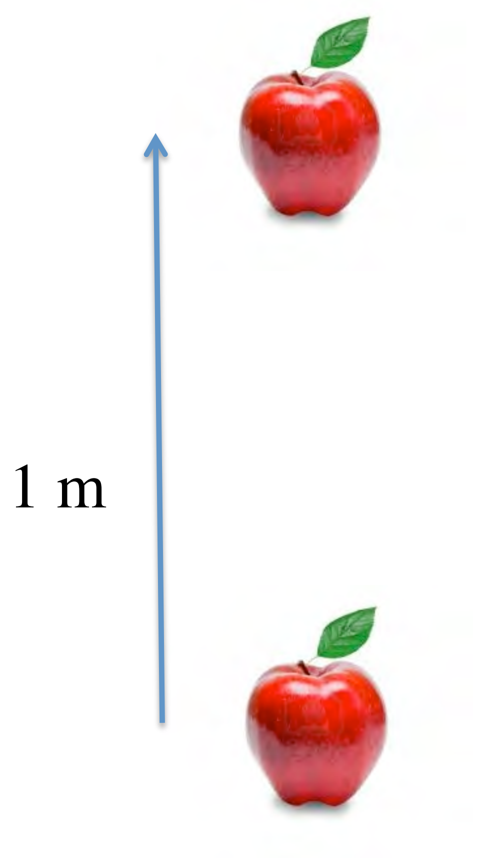
Unit of measure: Joule = 1 J = 1 N x 1 m

Power = Energy /time



Task	Power (W)
Average power of a Boing 747 airplane	10^8
Full power aircraft fighter	10^6
Full power car engine	10^5
Operate a microwave oven	10^3
Being alive for an average adult human	10^2
Brain functioning for an average human	10
mobile phone calling	1
Emission of a standard WI-FI router	10^{-1}
Functioning of a LED light	10^{-2}
Functioning of a miniature FM receiver	10^{-3}
Functioning o a wireless sensor node	10^{-4}
Low power radio module	10^{-5}
Functioning of a quartz wristwatch	10^{-6}
Operation of a quartz oscillator	10^{-7}
Sleep mode of a microcontroller	10^{-8}
1 bit information erasure at room T (min)	10^{-21}

Energy is a property of a physical system that thanks to this property can do some work.



It goes also the other way around…

By doing work I can increase the energy content of a physical system

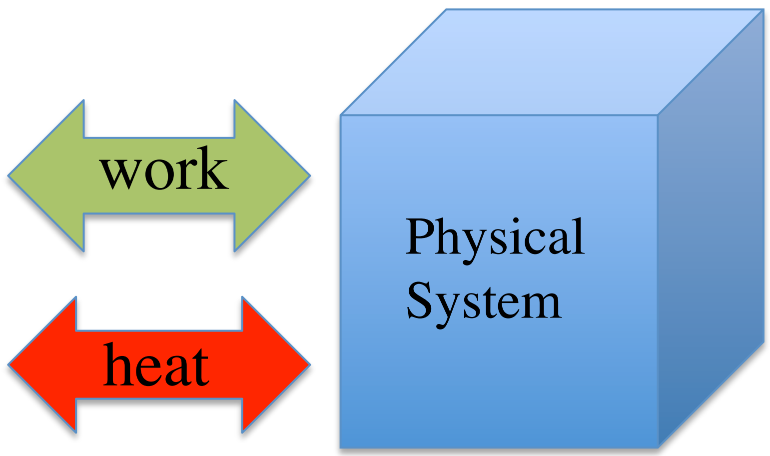
Is there any other way to change the energy content of a system?

Yes!

We can warm it up!

Energy

The energy content of a system can be changed via exchanges of work and heat



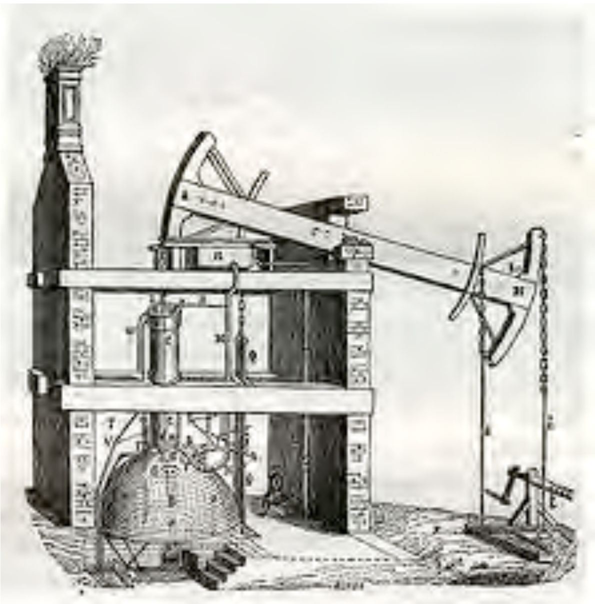
but there are some limitations…

Energy

Energy is a property of physical systems that can be used to perform work and usually comes inside physical objects like a **hot gas** or a **gasoline tank**.

Thinking about it we can ask questions like:

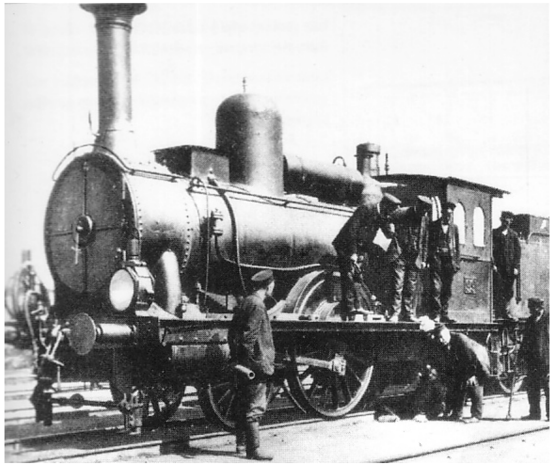
- how can we make the energy contained in a litre of gasoline to push forward a car
- how can we use the heat produced by burning coal to make the train run?



Questions like these were at the very base of the activities performed in the early seventeen hundreds by the first inventors of the so-called thermal machines.
People like **Thomas Newcomen** (1664-1729) who built the first practical steam engine for pumping water and **James Watt** (1736-1819) who few decades after proposed an improved version of the same machine.

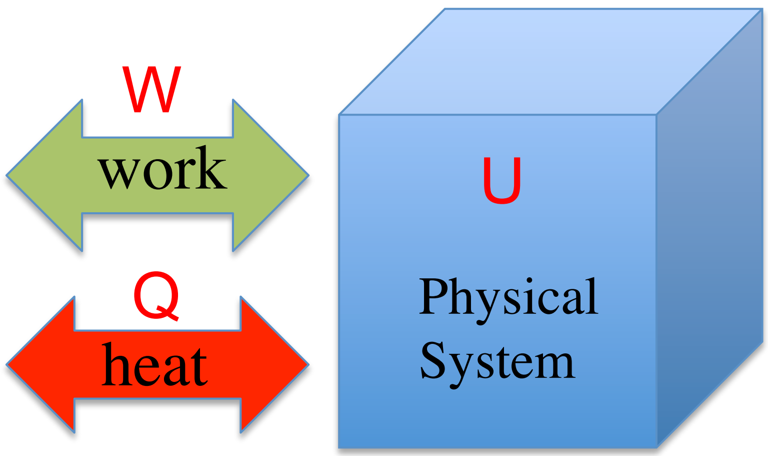
Thermal Machines

It is thanks to the work of scientists like Sadi Carnot (1796-1832) and subsequently of Émile Clapeyron (1799 - 1864), Rudolf Clausius (1822 - 1888) and William Thomson (Lord Kelvin) (1824 – 1907) that studies on the efficiency of these machines aimed at transforming heat (just a form of energy) into work brought us the notion of entropy and the laws of thermodynamics.



These laws do not tell us much about what energy is but they are very good in ruling what can we do and what we cannot do with energy. Let's briefly review them.

The first law of thermodynamics states that the total energy of an isolated physical system is conserved during any transformation the system can go through.



$$\Delta U = Q - W$$

Change in internal energy; Heat added to the system; Work done by the system

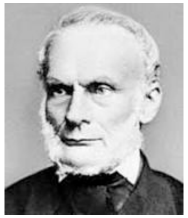
It was initially formulated in this form by Julius Robert von Mayer (1814 - 1878) and subsequently reviewed by James Prescot Joule (1818-1889) and Hermann Ludwig Ferdinand von Helmholtz (1821-1894).

"Now, in the *second* law of thermodynamics . . . "

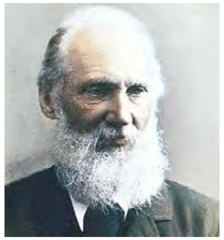
The second law states that there are limitations to how much work we can get from a given amount of energy present in the form of heat.

The second law states that there are limitations to how much work we can get from a given amount of energy present in the form of heat.

There exist different formulations that are all equivalent.
The two most popular are ascribed to Clausius and Kelvin:



Clausius formulation: "**No process is possible whose sole result is the transfer of heat from a body of lower temperature to a body of higher temperature**".

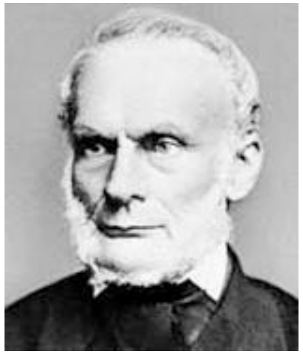


Kelvin formulation: "**No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work**".

An important consequence of the second law is that there is a limit to **the efficiency of a thermal machine**. This limit was discovered by Sadi Carnot in 1824 when he was only 28. He introduced the concept of thermal machine, generalizing the concept popular at that time of “steam engine”, and showing that the efficiency of any thermal machine operating between two temperatures is bounded by a quantity that is a function of the two temperatures only.

Few years after the work of Carnot, Clausius used this result to introduce a quantity that is useful in describing how much heat can be changed into work during a transformation. He proposed the name “**entropy**” for his quantity.

ENTROPY



Clausius proved a theorem that states that during a **cyclic transformation**, if you do the transformation carefully enough not to loose any energy in other ways (like friction), then **the sum of the heat exchanged with the external divided by the temperature at which the exchange occurs is zero:**

$$\oint \frac{dQ}{T} = 0$$

This is equivalent to say that it exists a state function *S* defined as

$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

If you are not careful enough and you loose energy during the transformation than the inequality holds instead:

$$\oint \frac{dQ}{T} \leq 0$$

A transformation like this is also called an *irreversible transformation*

$$S_B - S_A \geq \int_{A\,irr}^{B} \frac{dQ}{T}$$

If we consider an infinitesimal transformation we have:

$$dS \geq \frac{dQ}{T} \quad \text{or} \quad TdS \geq dQ$$

where the equal sign hold during a reversible transformation only.

Spontaneous transformation

We call a transformation spontaneous if it can happen without any external work.

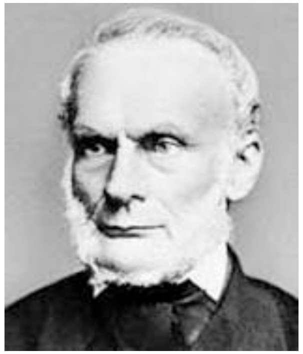
Let's suppose that we have a transformation where no heat nor work is exchanged.
It is called an adiabatic spontaneous transformation.
In this case we have:

$$dS > 0$$

Thus we conclude that during a spontaneous adiabatic transformation (i.e. without external work nor heat) the **entropy always increases**.

What happens if we have exchange of heat?

From Clausius we know that, for a reversible transformation we have:



$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

The quantity of heat Q that appears in the Clausius equation is the amount of energy that goes into the increase of entropy.

For an infinitesimal transformation: $TdS = dQ$

It is useful to interpret the quantity ***TdS* as the amount of heat** (meaning thermal energy**) that cannot be used to produce work**.

In other words during a transformation, even if we are carefully enough not to waste energy in other ways, we cannot use all the energy we have to do useful work. Part of this energy will go into producing a change of the system entropy.

If we are not carefully enough the situation is even worst and we get even less work.

Free energy

The concept of Free energy was proposed by Helmholtz in the form:

$$F = U - TS$$

The free energy F measures the maximum amount of energy that we can use when we have available the internal energy U of a system.

Equilibrium

A system can undergo a spontaneous transformation until it reaches a state of equilibrium.

What is it?

The importance of the notion of relaxation time

Equilibrium

The importance of the notion of relaxation time

Equilibrium (within a given relaxation time) is the competition of the tendency of:

Energy to reach a minimum — Entropy to reach a maximum

$$F = U - T S$$

Minimum free energy

Summary

Energy

Capability of doing WORK… WORK = FORCE x Displacement
Energy can be **changed** through flux of **work** and **heat**.

Energy is conserved (First Principle)

Entropy

Measures the capability of change…

Entropy increases in spontaneous transf. (Second Principle)

Equilibrium is the competition of the tendency of Energy to reach a minimum and Entropy to reach a maximum (minimum Gibbs free energy).

$$F = U - T S$$

Energy transformations at micro scales

The microscopic interpretation of heat: the kinetic theory

In approximately 50 BCE, the Roman philosopher Lucretius proposed that apparently static macroscopic bodies were composed on a small scale of rapidly moving atoms all bouncing off each other.

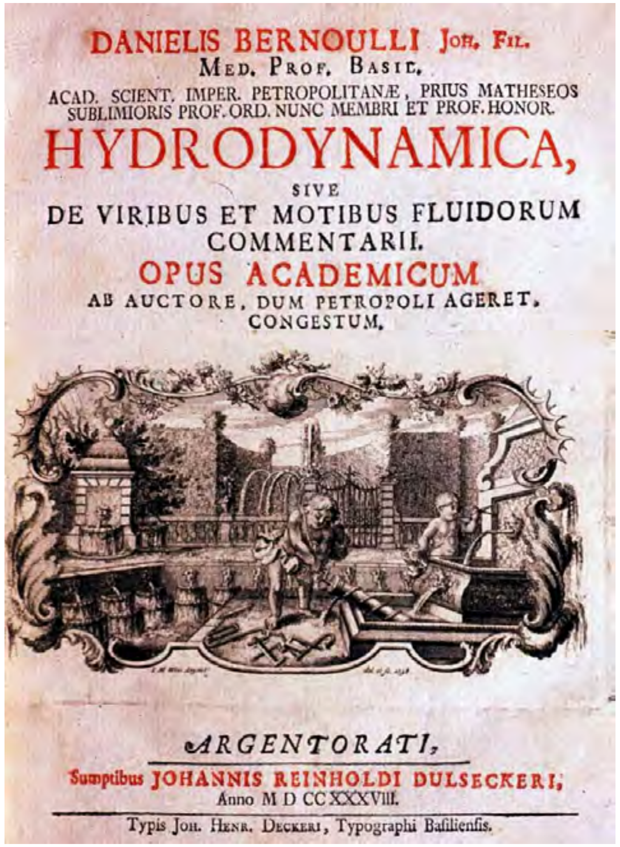
This Epicurean atomistic point of view was rarely considered in the subsequent centuries, when Aristotlean ideas were dominant.

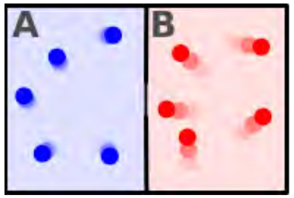
The microscopic interpretation of heat: the kinetic theory

In 1738 **Daniel Bernoulli** published Hydrodynamica, which laid the basis for the kinetic theory of gases.

Physicist James Clerk Maxwell, in his 1871 classic Theory of Heat, was one of many who began to build on the already established idea that **heat has something to do with matter in motion.**

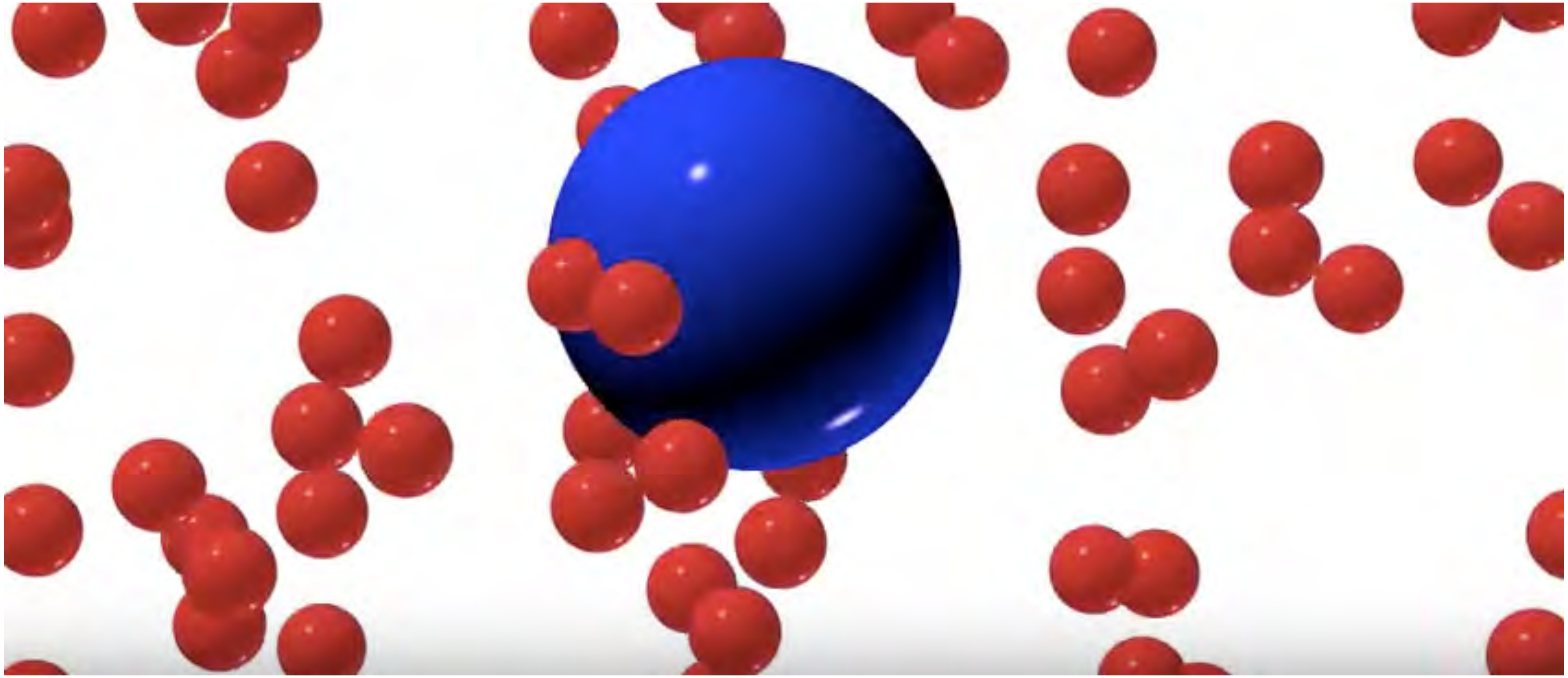
This was the same idea put forth by Benjamin Thompson in 1798, who said he was only following up on the work of many others.





$T_1 < T_2$

Brownian motion

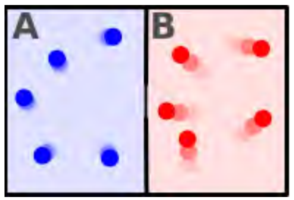


https://www.youtube.com/watch?v=iTbEmvdzvxg

The microscopic interpretation of heat: the kinetic theory

The theory for ideal gases makes the following assumptions:

- The gas consists of very small particles known as molecules. The average distance separating the gas particles is large compared to their size.
- These particles have the same mass.
- The number of molecules is so large that statistical treatment can be applied.
- These molecules are in constant, random, and rapid motion.
- The rapidly moving particles constantly collide among themselves and with the walls of the container. All these collisions are perfectly elastic.
- Except during collisions, the interactions among molecules are negligible. (That is, they exert no forces on one another.)



$$T_1 < T_2$$

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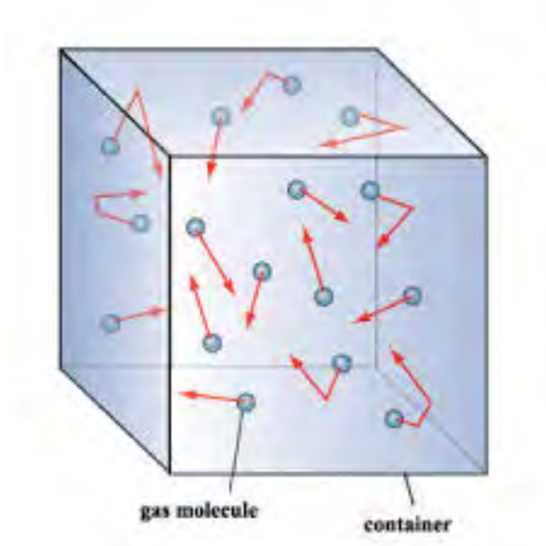
Pressure

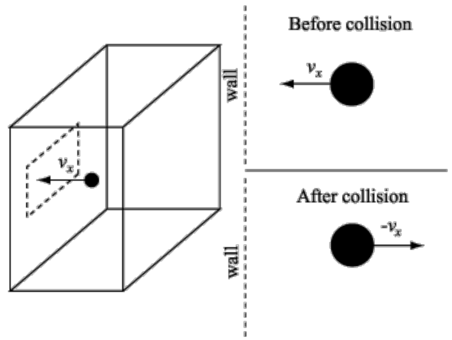


Home work

Pressure is explained by kinetic theory as arising from the force exerted by molecules or atoms impacting on the walls of a container.

Consider a gas of N molecules, each of mass m, enclosed in a cuboidal container of volume $V=L^3$.

Pressure



Home work

When a gas molecule collides with the wall of the container perpendicular to the x coordinate axis and bounces off in the opposite direction with the same speed (an elastic collision), then the momentum lost by the particle and gained by the wall is:

$$\Delta p = 2\ m\ v$$

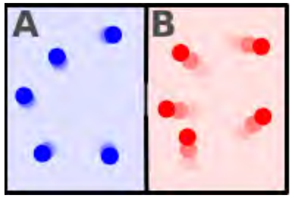
The particle impacts one specific side wall once every $\Delta t = 2L/v$ (where L is the distance between opposite walls).

The force due to this particle is: $F = \Delta p/\Delta t = m\ v^2/L$

The total force on the wall is $F = Nm\ v^2/3L$ (averaging on the 3 directions)

And thus the pressure is $$P = \frac{F}{L^2} = \frac{Nm\overline{v^2}}{3V}$$

Temperature



$T_1 < T_2$

From before we have

$$PV = \frac{Nm\overline{v^2}}{3}$$

By comparing with the ideal gas law: $PV = Nk_BT$

we have

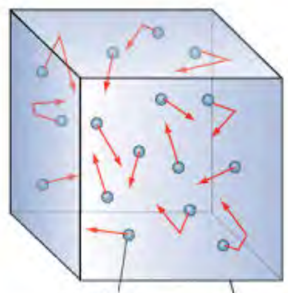
$$k_BT = \frac{m\overline{v^2}}{3},$$

and thus:

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT.$$

Than links average kinetic energy of a molecule with temperature.

The microscopic interpretation of heat: the kinetic theory



OK fine.

But what about Entropy?

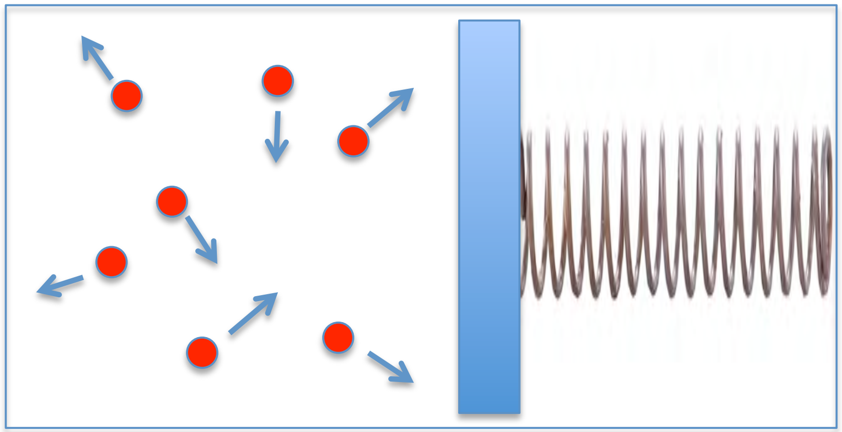
Energy and Entropy: the microscopic interpretation

In general the entropy remained an obscure quantity whose physical sense was (and somehow still is) difficult to grasp.

It was the work of Ludwing Boltzmann (1844 – 1906) that shed some light on the microscopic interpretation of the second law (and thus the entropy).

To grasp the meaning of entropy at small scales…

The ideal world of Boltzmann is made by physical systems constituted by many small parts represented by colliding small spheres



Each sphere has the same mass *m* and velocity *v*

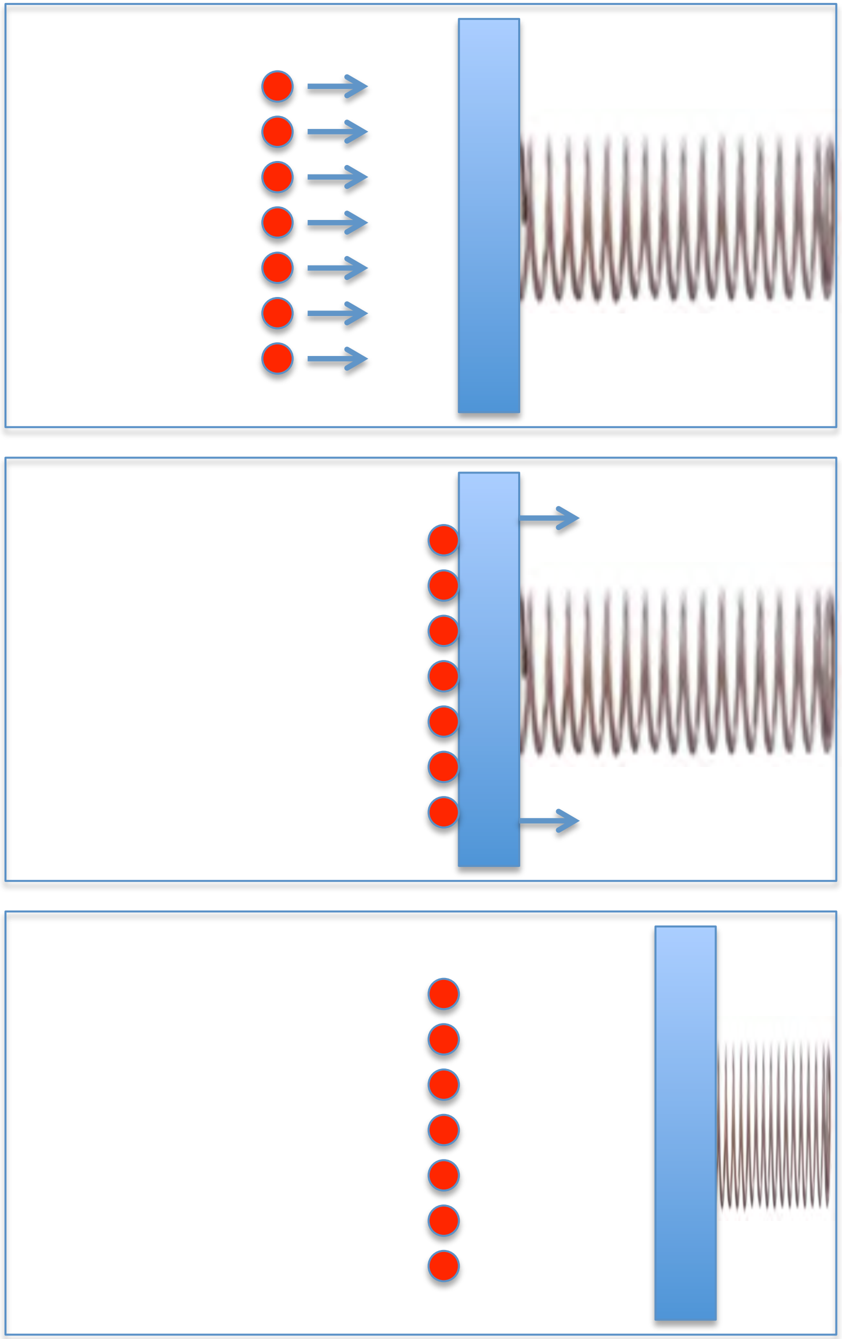
Consider the two cases…

First case

Let's suppose that these particles are contained in a box that has a moving set of mass $M = Nm$. The set is connected to a spring of elastic constant k, as in the figure, and is at rest.

If all the particles have the same velocity v and collide perpendicularly with the moving set at the same time, they will exchange velocity with the set. This will compress the spring up to an extent x_1 such that

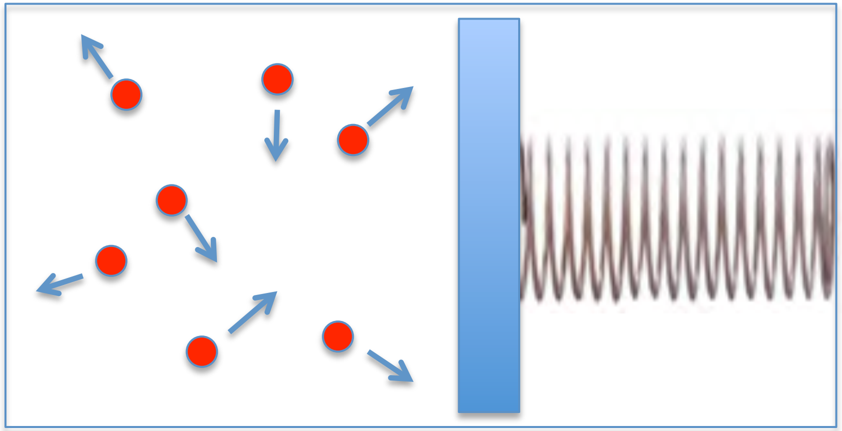
$$\frac{1}{2} M v^2 = \frac{1}{2} k x_1^2 = U$$

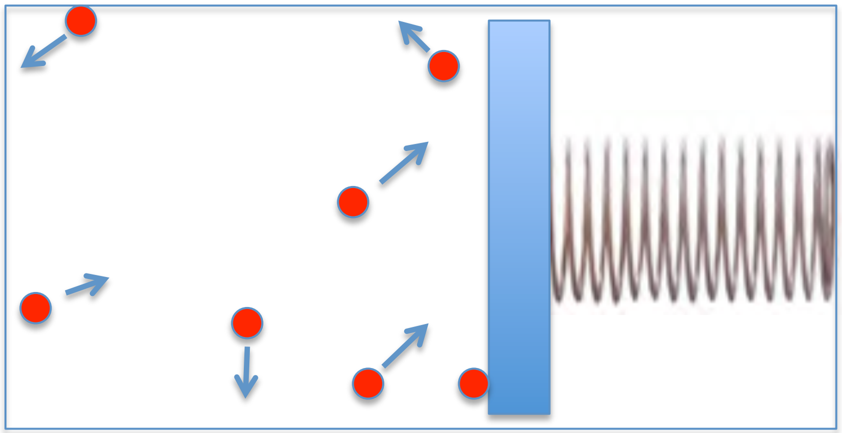


We can always recover the potential energy U when we desire and use it to perform work. In this case we can completely transform the energy of the gas particle into work.

Second case

What is on the contrary the most probable configuration for the particle in the gas? Based on our experience (and on some common sense as well) it is the configuration in which all the particles, although each with the same velocity v, are moving with random direction in the box.





The **energy of the gas is still the same (so is its temperature T)** but in this case the set will be subjected at random motion with an average compression of the spring such that its average energy is U/N.

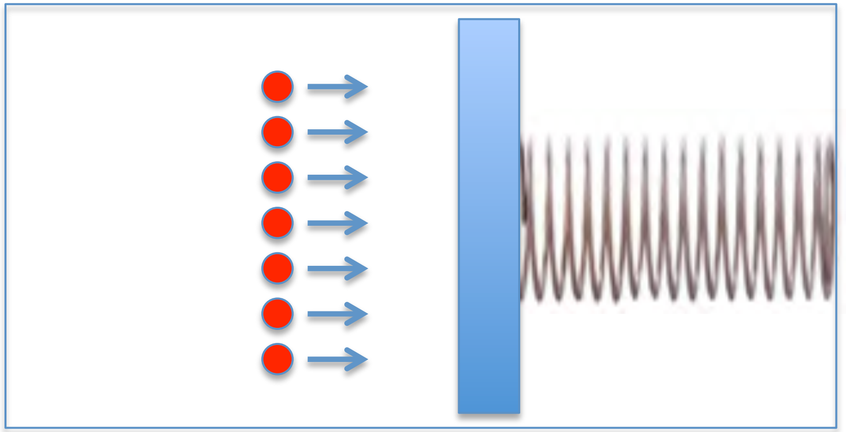
This is also the maximum work that we can recover from the potential energy of the movable set.

According to the definition of Free energy, the quantity that limits our capability of performing work is the entropy.

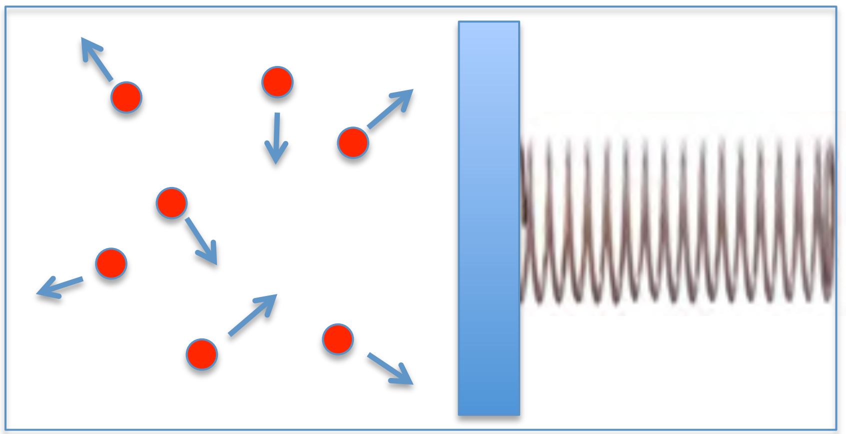
Thus the systems that have the smaller entropy have the larger capability of performing work.

Accordingly we can use the entropy to put a label on the energetic content of a system.

Two systems may have the same energy but the system that has the lower entropy will have the "most useful" energy.



low entropy



high entropy

The microscopic perspective

This example helped us to understand how energy and entropy are connected to the microscopic properties of the physical systems.

In the simple case of an ideal gas, the system energy is nothing else than the sum of all the kinetic energies of the single particles.
We can say that the energy is associated with "how much" the particles move.

On the other hand we have seen that there is also a "quality" of the motion of the particles that is relevant for the entropy.
We can say that the entropy is associated with "the way" the particles moves.

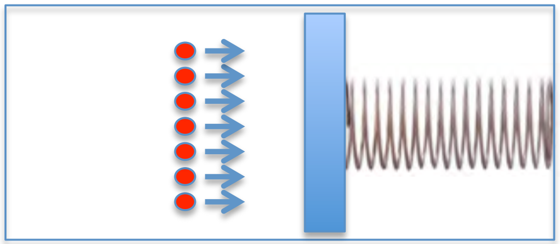
The microscopic perspective

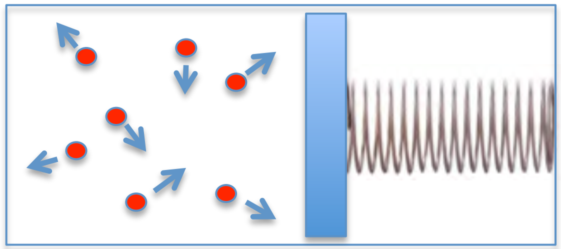
The entropy is associated with "the way" the particles moves.

This concept of "way of moving" was made clear by Boltzmann at the end of 1800, who proposed for the entropy the following definition:

$$S = K_B \log W$$

where K_B is the famous Boltzmann constant and W is also called the "number of configurations" and represents the number of ways we can arrange all the particles in the system without changing its macroscopic properties.





Few ways = low entropy

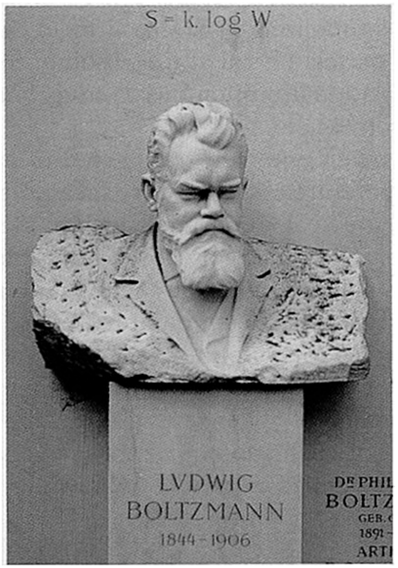
many ways = high entropy

The microscopic perspective

The second principle

In a spontaneous transformation the entropy always increases

If a system can be in a number of different states, all compatible with the conservation laws, then it will evolve in order to attain the equilibrium condition identified with the **most probable state** among all the possible states it can be in.



What about the Friction?

During an irreversible transformation the entropy always increase more that what was expected, due to the Clausius equality that becomes ***inequality***:.

$$S_B - S_A \geq \int_{A\,irr}^{B} \frac{dQ}{T}$$

Why is that? The answer is that in addition to the *physiological* increase there is an extra contribution due to the *dissipative effect* of the non-equilibrium processes. With *dissipative effect* we intend a way in which some low-entropy energy is changed into high-entropy energy. A typical example of dissipative process is friction.

FRICTION

How can we describe friction?

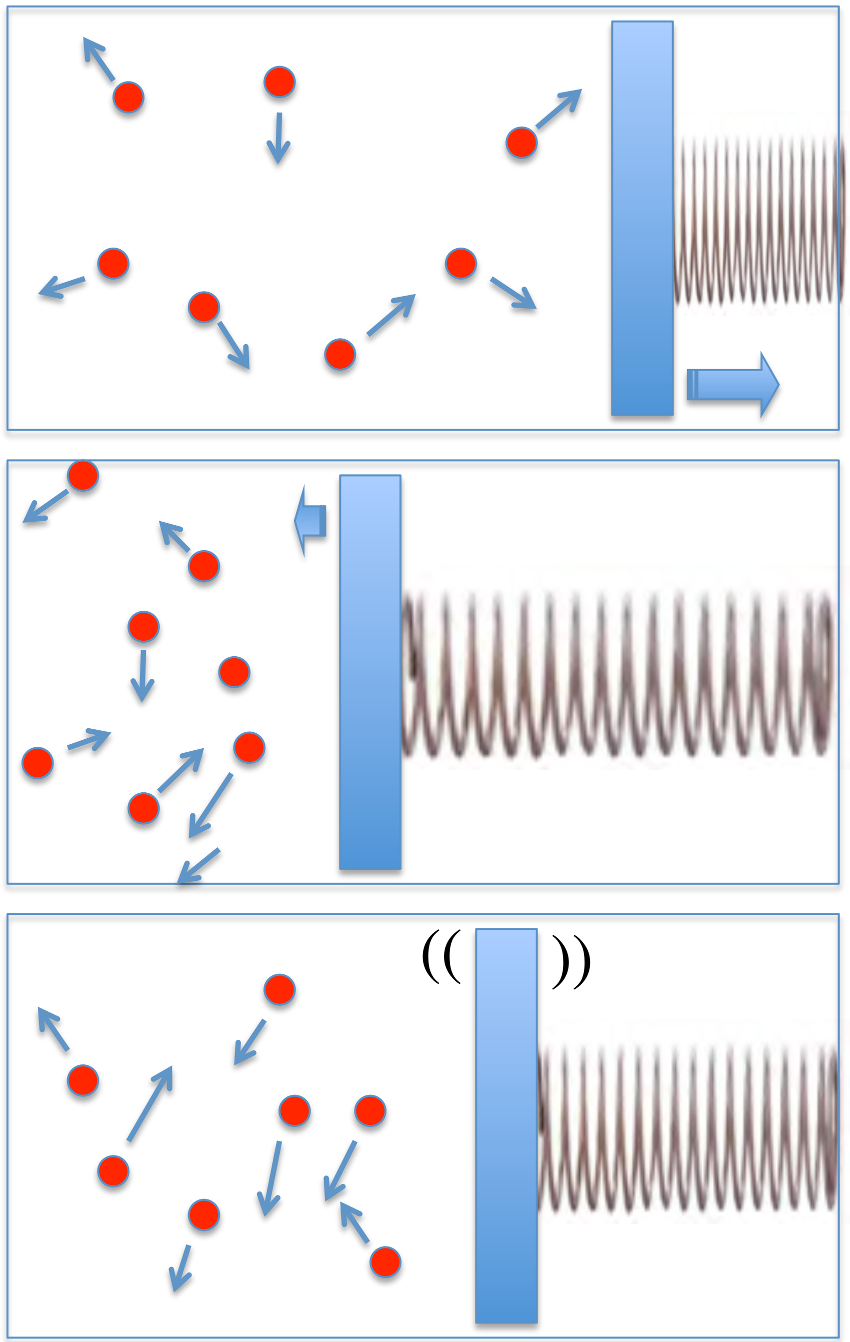
Consider the two cases…

First case

We compress the spring to some extent and then we release the compression leaving it free to oscillate.

After few oscillations we observe that the oscillation amplitude decreases as a consequence of what we call the friction (viscous damping force) action due to the presence of the gas.
The decrease ceases when the oscillation amplitude reaches a certain equilibrium value and after that it remains constant (on average).

Some energy has been dissipated into heat.

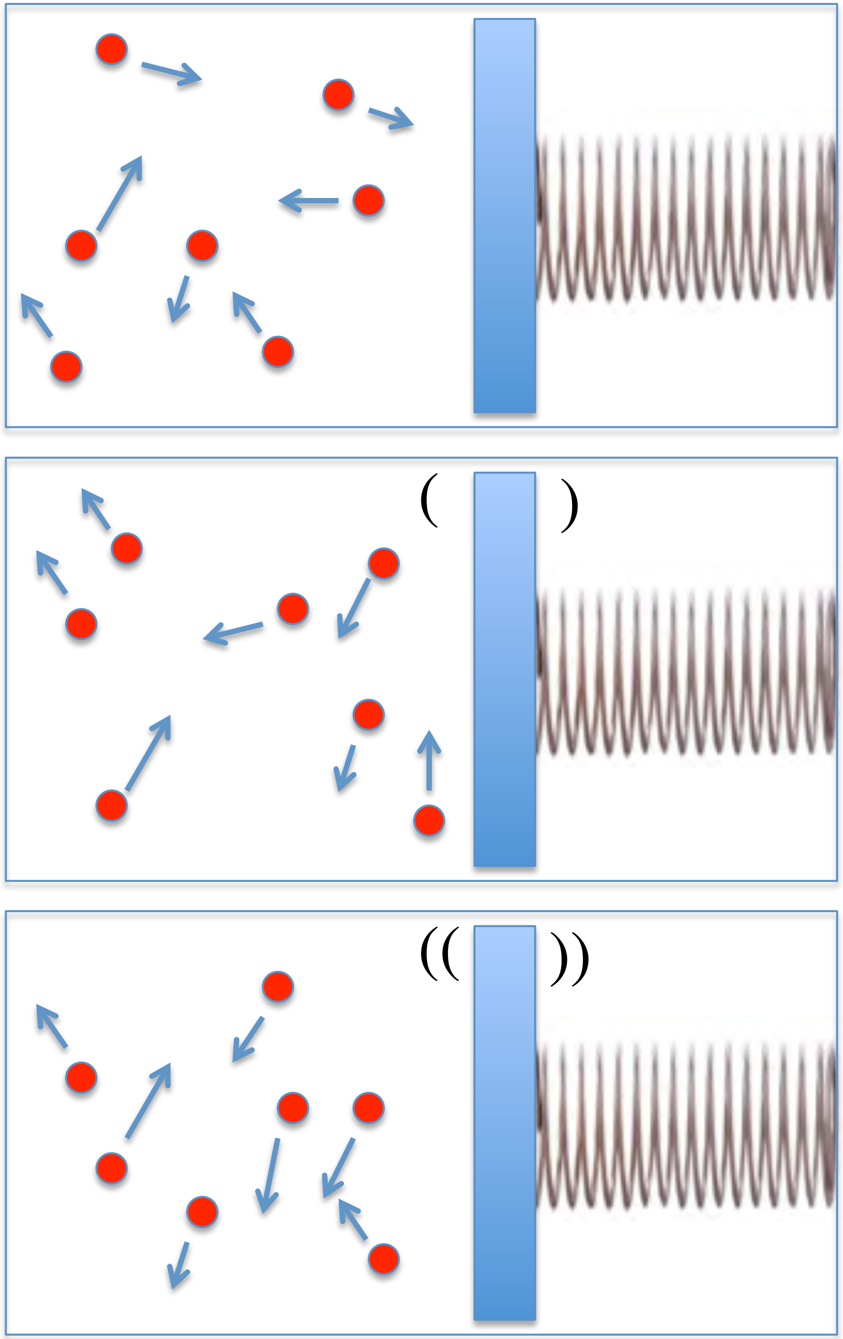


Second case

We now start with the movable set at rest and leave it free.

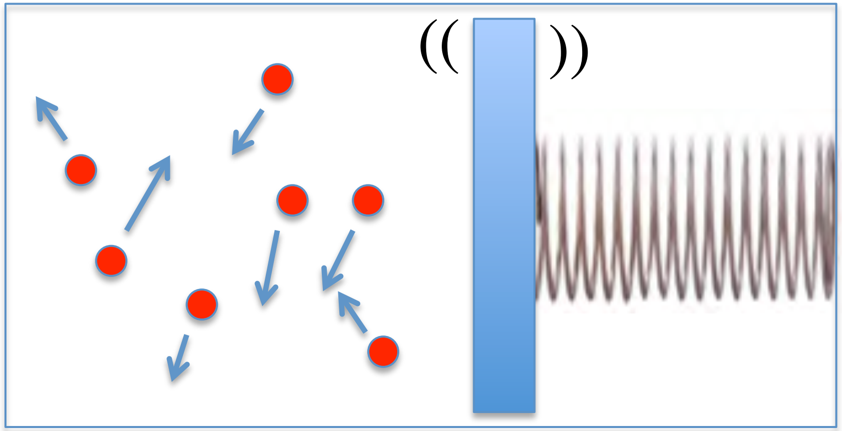
After few seconds we will see that the set starts to move with increasing oscillation amplitude that soon reaches **an equilibrium condition** at the very same value (on average) of the first case.

In both cases the two different roles of damping-force and pushing-force has been played by the gas.
This fact led to think that there must be a **connection between the process of dissipating energy** (a typical irreversible, i.e. non-equilibrium process) and the process of **fluctuating at equilibrium** with the gas.



Fluctuation-Dissipation connection

This fact led to think that there must be a **connection between the process of dissipating energy** (a typical irreversible, i.e. non-equilibrium process) and the process of **fluctuating at equilibrium** with the gas.



In order to unveil such a link we need to introduce a more formal description of the dynamics of the movable set.

This problem has been addressed and solved by Albert Einstein (1879 - 1955) in his 1905 discussion of the Brownian motion and subsequently by **Paul Langevin** (1872 - 1946) who proposed the following equation:

$$m\ddot{x} = -m\gamma\dot{x} - \frac{dU}{dx} + \xi(t)$$

Fluctuation-Dissipation connection

$$m\ddot{x} = - m\gamma\dot{x} - \frac{dU}{dx} + \xi(t)$$

x(t) is the random force that accounts for the incessant impact of the gas particles on the set, assumed with zero mean, Gaussian distributed and with a flat spectrum or, delta-correlated in time (white noise assumption):

$$\langle\xi(t_1)\xi(t_2)\rangle = 2\pi\, G_R\delta(t_1 - t_2)$$

G_R accounts for the fluctuation intensity. There must be a connection with the dissipation γ.

This relation has been established within the linear response theory (that satisfies the equipartition of the energy among all the degrees of freedom) initially by **Harry Theodor Nyquist** (1889 - 1976) in 1928, and demonstrated by **Callen and Welton** in 1951.

$$G_R = \frac{mK_BT}{\pi}\gamma$$

Fluctuation-Dissipation Theorem FDT

Why is FDT important?

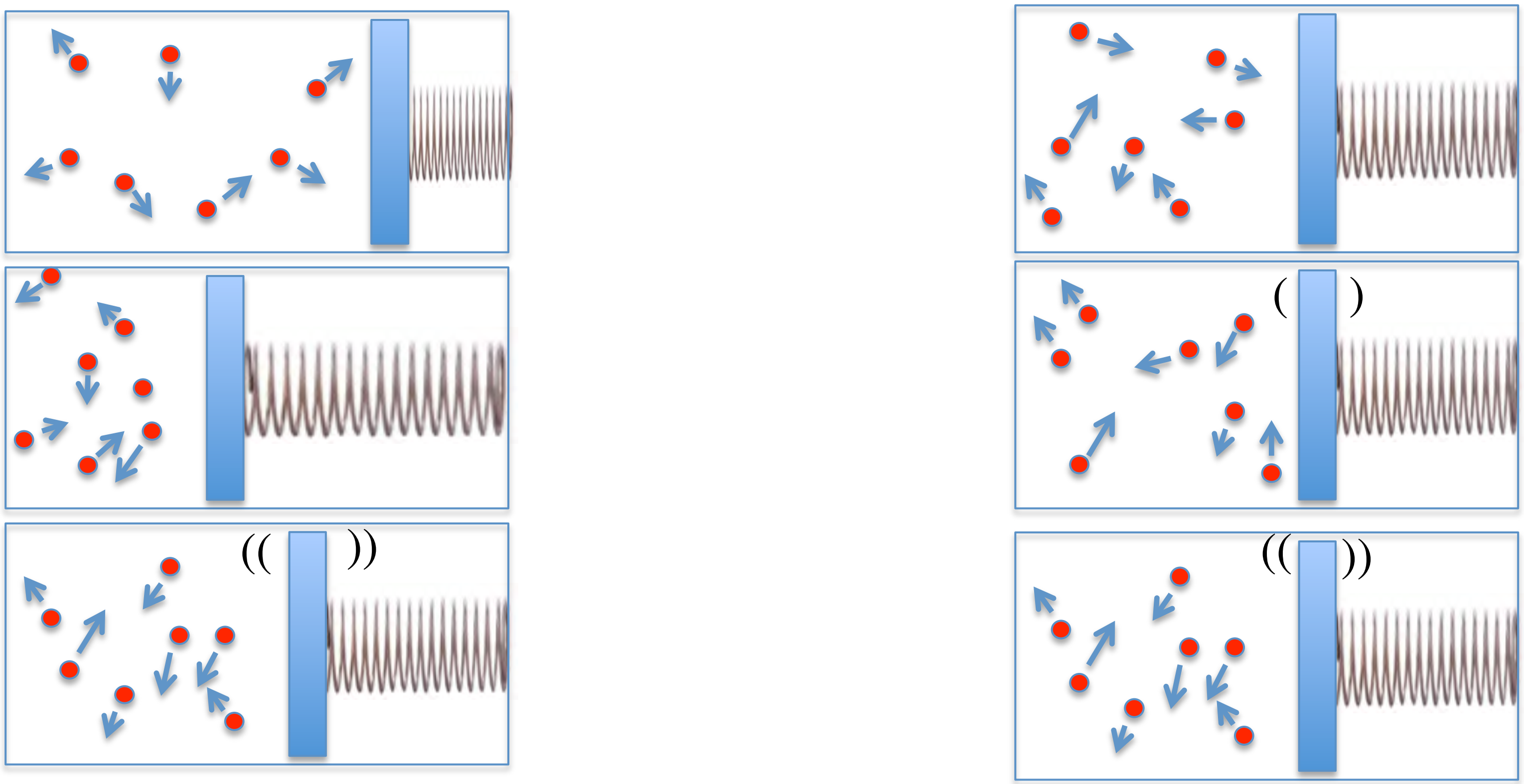
It is important because it represent an ideal bridge that connects:

the equilibrium properties of our thermodynamic system
(represented by the amplitude and character of the fluctuations)

with

the non-equilibrium properties
(represented here by the dissipative phenomena due to the presence of the friction).

ENTROPY production – FLUCTUATION CONNECTION



dissipative properties = the capacity to produce entropy, are intrinsically connected to the equilibrium fluctuations.

Summary

- All matter is made by particles
- We can say that the energy is associated with “how much” the particles move.
- We can say that the entropy is associated with “the way” the particles moves.
- Entropy according to Boltzmann: $S = K_B \log W$
- *W* is the number of configurations

Second Principle: “In a spontaneous transformation the entropy always increases“ Is equivalent to say that “If a system can be in a number of different states, all compatible with the conservation laws, then it will evolve in order to attain the equilibrium condition identified with the **most probable state** among all the possible states it can be in. ”

Friction: there is a connection between the process of dissipating energy (a typical irreversible, i.e. non-equilibrium process) and the process of fluctuating at equilibrium with the gas: the Fluctuation-Dissipation Theorem.

To learn more:

Energy Management at the Nanoscale
L. Gammaitoni

in the book "ICT - Energy - Concepts Towards Zero - Power Information and Communication Technology” InTech, February 2, 2014

https://www.intechopen.com/books/ict-energy-concepts-towards-zero-power-information-and-communication-technology/energy-management-at-the-nanoscale